SUPPLEMENTARY MATERIAL

1 Supplementary Tables

Table S1. Estimated reference values for the cooperative regime, $\hat{x}_{CC}^{CR}$

α	μ = 0.05	μ = 0.01	μ = 0.001
10	39.7 (6.8)%	59.9 (5.8)%	65.5 (5.6)%
11	45.1 (6.2)%	65.0 (5.3)%	69.2 (5.6)%
12	50.0 (5.4)%	68.0 (5.1)%	73.2 (5.3)%
13	53.5 (5.4)%	71.0 (4.7)%	75.6 (4.9)%
14	56.6 (4.6)%	73.4 (4.5)%	78.4 (5.4)%
15	59.0 (4.6)%	75.1 (4.2)%	79.7 (4.3)%
20	66.2 (3.9)%	81.5 (3.7)%	85.0 (3.8)%
25	71.1 (3.6)%	84.7 (3.5)%	89.1 (3.4)%
30	73.2 (3.3)%	87.1 (2.9)%	91.1 (3.1)%
40	76.4 (3.0)%	89.4 (2.5)%	93.4 (2.6)%
50	78.7 (2.7)%	90.8 (2.3)%	95.4 (2.7)%
100	82.8 (2.4)%	93.8 (1.7)%	97.6 (1.7)%

Estimated values for the average fraction of CC outcomes $\hat{x}_{CC}^{CR}$ that characterizes the cooperative regime. To calculate $\hat{x}_{CC}^{CR}$ we simulate the ergodic process until we obtain a series of 10^5 consecutive observations such that at least 99% of them are within the band defined by $|\, x_{CC} - \hat{x}_{CC}^{CR} \,| \leq \gamma$, where $\hat{x}_{CC}^{CR}$ is the average value of x_{CC} in the series, provided that $\hat{x}_{CC}^{CR}$ > 0.3. For α > 15 we took γ = 0.1; for $\alpha \leq 15$, γ = 0.2. For the particular case α = 10, given the lesser persistence of the cooperative regime, we required 25 000 consecutive observations. Table S2 shows that the value thus computed for each parameter configuration effectively anchors the level of cooperation observed in the cooperative regime (i.e. whenever the simulation run displays a sizable level of cooperation). The values in brackets show the standard deviation of x_{CC} in the series. Parameterization: N = 1000, T = 4, R = 3, P = 1, S = 0.

Table S2. Expected Lifespan, Non-cooperative Regime (NCR) and Cooperative Regime (CR)

α	$\hat{x}_{CC}^{CR}$ (%)	*NCR (%):* $x_{DD} \geq 0.8$	*CR (%):* $\lvert x_{CC} - \hat{x}_{CC}^{CR} \rvert \leq 0.1$	*Within regimes (%)*	*CR to NCR*	*NCR to CR*
5	-	100.0 (0.1)	0.0 (0.0)	100.0	0	0
10	*39.7%*	99.9 (0.5)	0.0 (0.0)	99.9	0	0
11	*45.1%*	99.1 (7.9)	0.6 (7.1)	99.7	0	1
12	*50.0%*	97.0 (15.7)	2.4 (14.4)	99.4	0	5
13	*53.5%*	90.4 (28.1)	8.4 (26.4)	98.8	0	20
14	*56.6%*	77.3 (40.5)	20.9 (38.7)	98.2	0	42
15	*59.0%*	64.2 (45.9)	33.4 (44.5)	97.6	0	80
20	*66.2%*	8.4 (24.6)	89.5 (25.5)	97.9	0	93
25	*71.1%*	0.3 (4.0)	98.8 (4.4)	99.1	0	7
30	*73.2%*	0.0 (0.0)	99.6 (0.5)	99.6	0	0
40	*76.4%*	0.0 (0.0)	99.9 (0.3)	99.9	0	0
50	*78.7%*	0.0 (0.0)	99.9 (0.3)	99.9	0	0
100	*82.7%*	0.0 (0.0)	100.0 (0.0)	100.0	0	0

Statistics in each row compiled over 10^3 simulation runs. Every run measured between periods $3{\cdot}10^3$ and 10^4. α: expected lifespan. $\hat{x}_{CC}^{CR}$: Reference level of cooperation for the cooperative regime (Table S1). *NCR*: visitation rate in the non-cooperative regime (fraction of periods with $x_{DD} \geq 0.8$). *CR*: visitation rate in the cooperative regime (fraction of periods with $\lvert x_{CC} - \hat{x}_{CC}^{CR} \rvert \leq 0.1$). The values in brackets show the standard deviation of the averages across runs. *CR to NCR* / *NCR to CR*: number of transitions from one regime to the other. Simulations start from random initial conditions. Parameterization: $N = 1000$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Table S3. Main strategies in the non-cooperative regime

α	*D_D_L*	*D_D_D*	*D_L_L*	*D_C_L*	*D_C_D*	*D_L_D*
5	40.7 (1.6) %	27.2 (1.3) %	6.8 (0.7) %	6.7 (0.7) %	6.6 (0.7) %	4.9 (0.4) %
10	49.3 (1.7) %	23.0 (1.3) %	5.6 (0.8) %	5.7 (0.8) %	5.7 (0.8) %	3.6 (0.4) %
11	50.1 (1.7) %	22.4 (1.3) %	5.5 (0.8) %	5.6 (0.8) %	5.7 (0.8) %	3.5 (0.4) %
12	50.7 (1.9) %	21.9 (1.4) %	5.5 (0.8) %	5.5 (0.8) %	5.6 (0.8) %	3.4 (0.4) %
13	51.4 (1.7) %	21.5 (1.4) %	5.3 (0.8) %	5.4 (0.8) %	5.7 (0.9) %	3.3 (0.4) %
14	51.9 (2.1) %	21.1 (1.6) %	5.3 (0.9) %	5.3 (0.9) %	5.7 (1.3) %	3.3 (0.4) %
15	52.5 (2.2) %	20.6 (1.7) %	5.2 (1.0) %	5.2 (0.9) %	5.7 (1.2) %	3.2 (0.5) %
20	54.0 (3.3) %	19.1 (3.0) %	5.1 (1.7) %	5.2 (1.5) %	5.6 (2.1) %	2.9 (0.7) %
25	54.9 (3.4) %	17.4 (2.4) %	4.9 (3.0) %	5.5 (1.5) %	5.7 (1.3) %	3.0 (0.7) %

Average values for the fraction of the main strategies in the non-cooperative regime between periods $3 \cdot 10^3$ and 10^4 in 1000 simulated runs of the process, with random initial conditions. The values in brackets show the standard deviation of the averages across runs. Parameterization: $N = 1000$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

Table S4. Main strategies in the cooperative regime

α	*C_C_L*	*C_C_D*	*D_D_L*	*D_L_L*	*D_C_L*
11	49.2 (2.9)%	5.9 (2.6)%	14.5 (1.5)%	8.1 (1.4)%	4.5 (0.3)%
12	54.2 (1.8)%	5.4 (2.2)%	12.7 (1.1)%	6.9 (0.8)%	4.1 (0.7)%
13	56.9 (6.2)%	5.3 (6.3)%	11.5 (1.5)%	6.3 (1.2)%	3.8 (0.7)%
14	59.3 (3.8)%	5.1 (3.4)%	10.5 (1.2)%	5.8 (1.1)%	3.7 (0.6)%
15	61.0 (5.8)%	5.2 (5.8)%	9.9 (1.2)%	5.3 (1.0)%	3.5 (0.6)%
20	66.5 (3.8)%	5.2 (3.5)%	7.6 (0.9)%	4.2 (0.6)%	2.9 (0.4)%
25	70.1 (1.4)%	4.7 (1.3)%	6.5 (0.7)%	3.6 (0.6)%	2.6 (0.4)%
30	72.0 (1.4)%	4.8 (1.2)%	5.8 (0.6)%	3.1 (0.5)%	2.4 (0.4)%
40	74.4 (1.7)%	4.9 (1.5)%	5.0 (0.6)%	2.7 (0.4)%	2.1 (0.3)%
50	75.7 (1.9)%	5.0 (1.7)%	4.6 (0.6)%	2.3 (0.4)%	1.9 (0.3)%
100	77.5 (3.5)%	6.5 (3.3)%	3.7 (0.6)%	1.7 (0.4)%	1.5 (0.3)%

Average values for the fraction of the main strategies in the cooperative regime between periods $3{\cdot}10^3$ and 10^4 in 1000 simulated runs of the process, with random initial conditions. The values in brackets show the standard deviation of the averages across runs. Parameterization: $N = 1000$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

2 Supplementary Figures

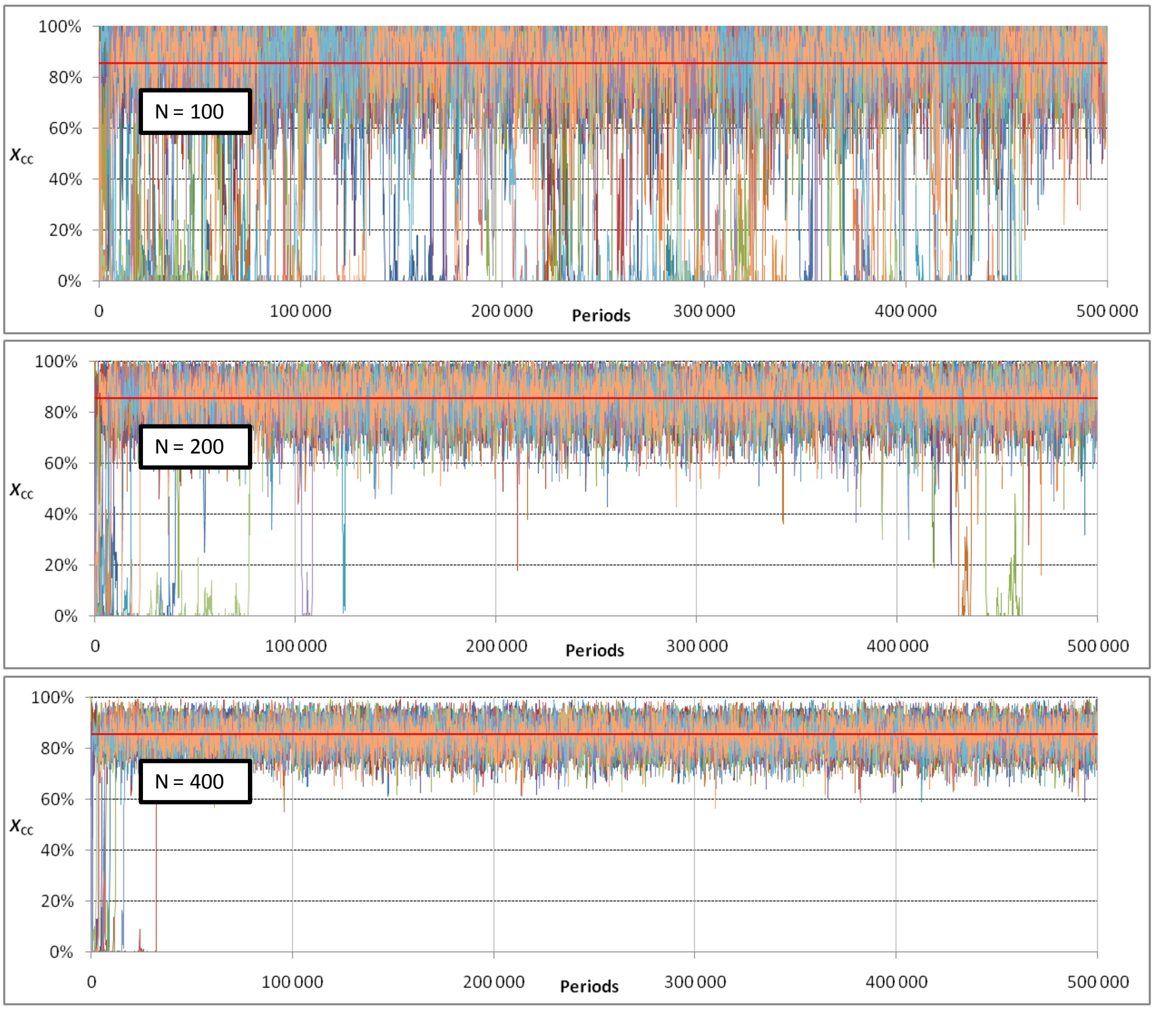

Fig. S1. Evolution of the percentage of CC outcomes (x_{CC}) for 18 different runs, each run starting with the whole population using the same strategy (one run for each of the 18 strategies). Each panel corresponds to a different population size: N = 100, N = 200 and N= 400; α = 25, μ = 0.01, T = 4, R = 3, P = 1, S = 0. The horizontal line at x_{CC} = 85.6% shows the stable level of cooperation in the mean-field approximation.

3. Supplementary Equations

3.1 Mean-field equations for the reduced system

Let x_{CC} be the share of pairwise interactions in which both parties play **C**, x_{DD} the share of such interactions in which both parties play **D**, and x_{CD} the share in which one of the individuals plays **C** and the other **D**. In the reduced system, which includes the strategy **C_C_L** and the three strategies **D_X_L**, the mean-field equations for x_{CC} , x_{DD} and x_{CD} in large populations ($N \rightarrow \infty$) with mutation rate μ are:

$$\dot{x}_{CC} = \frac{y_C^2}{y} + (1-\beta)^2 x_{CC} - x_{CC}$$

$$\dot{x}_{DD} = \frac{y_D^2}{y} - x_{DD}$$

$$\dot{x}_{CD} = 2\frac{y_C\, y_D}{y} - x_{CD}$$

where β is the death probability. Thus $(1 - \beta)^2$ is the probability that a pair will stay together one more period, and

$$y = 1 - (1-\beta)^2 x_{CC}$$

is the inflow to the pool of singles, composed of subflows y_C (which is the inflow of **C_C_L** singles) and y_D (which is the inflow of **D_X_L** singles):

$$y_C = \beta\left[(1-\mu)\frac{\pi_C}{\pi_C + \pi_D} + \frac{\mu}{4}\right] + (1-\beta)(\beta x_{CC} + \frac{x_{CD}}{2})$$

$$y_D = \beta\left[(1-\mu)\frac{\pi_D}{\pi_C + \pi_D} + \frac{3\mu}{4}\right] + (1-\beta)(x_{DD} + \frac{x_{CD}}{2})$$

where

$$\pi_C = R\, x_{CC} + S\frac{x_{CD}}{2}$$

and $\pi_D = P\,x_{DD} + T\,\frac{x_{CD}}{2}$

are the payoffs associated with **C**-players and **D**-players respectively.

It is easily verified that the mean-field equations leave the sum of interaction shares constant and all interaction shares non-negative:

$$\dot{x}_{CC} + \dot{x}_{DD} + \dot{x}_{CD} = \frac{(y_C + y_D)^2}{y} + (1-\beta)^2 - 1 = y - [1-(1-\beta)^2] = y - y = 0$$

Note: The reduced system considers the combination of the strategy **C_C_L** with the three strategies **D_X_L**. In terms of outcomes (x_{CC} , x_{DD} and x_{CD}), there are also other subsets of strategies (e.g. **C_C_D** with **D_L_L**) which correspond to the same Markov process (adapting the mutation term to the number of strategies) and which, consequently, present the same mean-field approximation and the same critical points.

3.2 Critical points for the particular case $T = 4$, $R = 3$, $P = 1$, $S = 0$, $\mu = 0$.

Let $\delta = (1-\beta)^2$ be the effective discount factor for cooperation. In our numerical example ($T = 4$, $R = 3$, $P = 1$, $S = 0$) the mean field with $\mu = 0$ admits two interior stationary states for all $\delta > 3/4$. In these, the share of cooperative interactions is

$$x^*_{CC} = 1 - \frac{1}{2\delta} \pm \sqrt{1 - \frac{3}{4\delta}}$$

The stationary point corresponding to the positive root is asymptotically stable (while the other is not) and the share of cooperative interactions in that point can be easily seen to converge to one as $\delta \rightarrow 1$.

3.3 Critical points for the general case, with $\mu = 0$

At the end of this section we prove that, for $\beta > 0$, the system

$$\dot{x}_{CC} = 0, \dot{x}_{DD} = 0, \dot{x}_{CD} = 0 \qquad [1]$$

is equivalent to the alternative system

$$\begin{cases} \dfrac{\pi_C}{\pi_C + \pi_D} = x_{CC} + \dfrac{x_{CD}}{2} \\ \dfrac{\pi_D}{\pi_C + \pi_D} = x_{DD} + \dfrac{x_{CD}}{2} \\ x_{CD}^2 = 4\, x_{DD}\, x_{CC}\, (1-\delta) \end{cases} \qquad [2]$$

where $\delta = (1-\beta)^2$. The third equation in [2] shows that the only critical points in the boundary of the unit simplex are $x_{CC} = 1$ and $x_{DD} = 1$, and that any other possible critical point must be interior. Now we look for the interior critical points ($x_{CD} > 0$).

From the first 2 equations of [2], one can obtain

$$x_{CC} + x_{DD} + x_{CD} = 1$$

and

$$(2\,x_{DD} + x_{CD})(2\,R\,x_{CC} + S\,x_{CD}) = (2\,x_{CC} + x_{CD})(2\,P\,x_{DD} + T\,x_{CD})$$

Expanding this last equation,

$$4(R-P)\,x_{CC}\,x_{DD} - 2(P-S)x_{DD}\,x_{CD} - 2(T-R)x_{CC}\,x_{CD} - (T-S)\,x_{CD}^2 = 0$$

and considering the third equation of [2] we may write:

$$(R-P)\frac{x_{CD}^2}{1-\delta} - 2(P-S)x_{DD}\,x_{CD} - 2(T-R)x_{CC}\,x_{CD} - (T-S)\,x_{CD}^2 = 0$$

or, given that $x_{CD} > 0$,

$$2(T-R)x_{CC} + 2(P-S)x_{DD} + (T - S - \frac{R-P}{1-\delta})x_{CD} = 0$$

Defining the normalized payoffs for reward (r) and for punishment (p) as

$$r = \frac{R - S}{T - S} \text{ and } p = \frac{P - S}{T - S}$$

we arrive at the system of three equations:

$$\begin{cases} 2(1-r)\,x_{CC} + 2\,p\,x_{DD} + (1 - \frac{r-p}{1-\delta})\,x_{CD} = 0 \\ x_{CD}^2 = 4\,x_{DD}\,x_{CC}\,(1-\delta) \\ x_{CC} + x_{DD} + x_{CD} = 1 \end{cases}$$

with 0 < x_{CC}, x_{DD}, x_{CD} < 1 and 0 < p < r < 1.

If $\delta \geq \delta_{Min} = \left(\sqrt{(1-r)(1-p)} + \sqrt{rp}\right)^2$, and we write $m = 1 - r - p$, the solution of this system for x_{CC} is

$$x_{CC} = \frac{m^2 + (1-2p)\delta\,[\delta - 2(1-r)] \quad \pm \quad |\,m - (1-2p)\delta\,|\,\sqrt{(m-\delta)^2 - 4rp\delta}}{2\delta[m^2(1-\delta) + \delta(r-p)^2]}$$

where the solution in which the root has a positive sign corresponds to the (unique) *stable* cooperative equilibrium.

Proof of the equivalence of the systems of equations [1] and [2]

a) $[1] \Rightarrow [2]$

Let $x_C = x_{CC} + \frac{x_{CD}}{2}$ be the fraction of **C**-players. Then

$$\dot{x}_C = \dot{x}_{CC} + \frac{\dot{x}_{CD}}{2} = \frac{y_C^2}{y} + (1-\beta)^2\,x_{CC} - x_{CC} + \frac{y_C\,y_D}{y} - \frac{x_{CD}}{2} =$$

$$= y_C + (1-\beta)^2\,x_{CC} - x_{CC} - \frac{x_{CD}}{2} =$$

$$= \beta\frac{\pi_C}{\pi_C + \pi_D} + (1-\beta)(\beta\,x_{CC} + \frac{x_{CD}}{2}) + (1-\beta)^2\,x_{CC} - x_{CC} - \frac{x_{CD}}{2} =$$

$$= \beta\frac{\pi_C}{\pi_C + \pi_D} - \beta(x_{CC} + \frac{x_{CD}}{2}) = 0$$

This equation is just the condition that the inflow of new **C**-players $\beta \frac{\pi_C}{\pi_C + \pi_D}$ and the outflow of dead **C**-players βx_C must be equal. Equivalently, for $x_D = x_{DD} + \frac{x_{CD}}{2}$ we obtain the second equation of [2].

The third equation of [2] is obtained from [1] using the relations

$$x_{CD}^2 = 4\frac{y_C^2}{y}\frac{y_D^2}{y}, \quad \frac{y_D^2}{y} = x_{DD} \text{ and } \quad \frac{y_C^2}{y} = x_{CC}[1-(1-\beta)^2]$$

b) $[2] \Rightarrow [1]$

The first 2 equations of [2] imply

$$\dot{x}_C = 0 \Rightarrow \dot{x}_{CC} + \frac{\dot{x}_{CD}}{2} = 0$$

$$\dot{x}_D = 0 \Rightarrow \dot{x}_{DD} + \frac{\dot{x}_{CD}}{2} = 0$$

$$y_D = (x_{DD} + \frac{x_{CD}}{2})$$

$$y_C = x_{CC}[1-(1-\beta)^2] + \frac{x_{CD}}{2}$$

$$x_{CC} + x_{DD} + x_{CD} = 1$$

Relying on these relations, we obtain

$$\dot{x}_{CD} = 2\frac{y_C\, y_D}{y} - x_{CD} = \frac{[2x_{CC}[1-(1-\beta)^2] + x_{CD}](2x_{DD} + x_{CD}) - 2y \cdot x_{CD}}{2y} =$$

$$= \frac{[2x_{CC}[1-(1-\beta)^2] + x_{CD}](2x_{DD} + x_{CD}) - 2\,[1 - x_{CC}(1-\beta)^2]x_{CD}}{2y} =$$

$$= \frac{4x_{CC}x_{DD}[1-(1-\beta)^2] + 2x_{CC}x_{CD}[1-(1-\beta)^2] + 2x_{CD}x_{DD} + x_{CD}^2 - 2[1 - x_{CC}(1-\beta)^2]x_{CD}}{2y} =$$

$$= \frac{4x_{CC}\,x_{DD}\,[1-(1-\beta)^2] + 2x_{CD}(x_{CC} + x_{DD} + x_{CD}) - x_{CD}^2 - 2x_{CD}}{2y} =$$

$$= \frac{4x_{CC}\,x_{DD}\,[1-(1-\beta)^2] - x_{CD}^2}{2y}$$

and considering the third equation in [2], we have [1].

4. Supplementary notes

4.1 Replication

All the simulations reported in this paper can be run using the available applets. Random initial conditions for the population are created by letting each individual take one of the 18 possible strategies with equal probability.

4.2 Costs of leaving

It is conceivable that, by penalizing strategies that "punish" defectors through dissociation, the existence of some additional cost when breaking a relationship could harm conditional dissociation and the establishment of a cooperative regime. However, computational studies in related frameworks (Vanberg & Congleton 1992; Aktipis 2004) show that cooperation can thrive with a costly exit option. This is also the case in our framework.

A natural way to model a cost of leaving is to assume that individuals do not immediately find a partner after splitting-up, but may spend some periods searching, during which they receive some low outside payoff instead. Consider the following extension of the conditional dissociation model: at the beginning of every time-step, every single (non-paired) player is selected to be paired with probability ϕ, and those who remain single receive an *outside payoff A* (if the number of players selected to be paired is odd, one of those selected players goes back to the pool of singles).

As a representative example, consider the following parameterization: N = 1000, μ = 0.05, T = 4, R = 3, P = 1, S = 0, A = 0, ϕ = 0.5. Comparing simulations of this model with those obtained for the original formulation with immediate re-matching (ϕ = 1), uncertain re-matching leads to a shorter minimum expected lifespan for the cooperative regime to appear (i.e., a lower δ_{Min}). And, for a given expected lifespan, it also presents an average level of cooperation in the cooperative regime (measured by the number of CC outcomes divided by the total number of outcomes in a given period) that is higher than in the original model. In order to understand this effect, note that the cost of a broken relationship falls predominantly upon those individuals who separate more often, and, in a cooperative regime supported by conditional dissociation, the proportion of separations is much higher among individuals who defect (for, typically, they leave or are abandoned after each interaction) than among individuals who cooperate (most of whom remain together in cooperating pairs, free from the additional costs of leaving).

5. Individual-based model with variable population size

In this section we consider an alternative model where, in every period, each individual breeds one offspring with probability β, and may die with the same probability β (both events being stochastically independent). Newborns cannot die in the time-step they are born, and they independently copy the decision rules of the individuals that were alive at the beginning of the current time-step, i.e. newborns cannot be copied. The intra-period sequence of events of this model is summarised in Fig. S2.

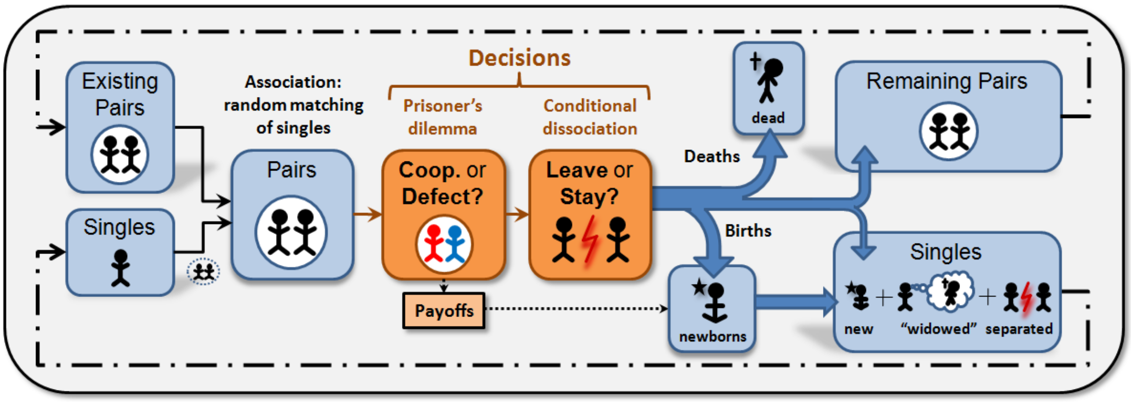

Fig. S2. Sequence of events within each time period. The "Remaining Pairs" and "Singles" at the end of a period are identical to the "Existing Pairs" and "Singles" in the next period.

Naturally, the population size N_t in this alternative model is variable. To be precise, both the number of births and the number of deaths in time-step t are binomial random variables with expected value $\beta \cdot N_t$, where N_t denotes the population size at the beginning of time-step t. Thus, for large populations, the population growth in time-step t, i.e. (N_{t+1} - N_t), is well approximated by a normal distribution with expected value 0. At those iterations of the model where N_t is odd, one randomly chosen individual remains unpaired and gets the same payoff as in the previous period.

5.1 Simulation results

The comparison between figures S3 and S4 below and figures 5 and 6 in the paper shows that the qualitative results obtained with the model where the population size is variable are identical to those obtained with the model where the population size is constant.

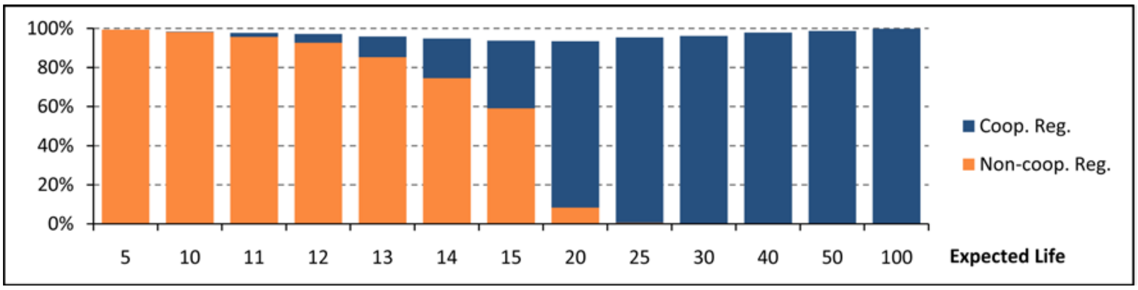

Fig. S3. Fraction of periods spent in the cooperative and in the non-cooperative regimes as a function of the expected life α in the model with variable population size. The values in each column are compiled over 10^3 simulation runs where the population has not extinguished at time-step 10^4. Every run measured between periods $3{\cdot}10^3$ and 10^4, with random initial conditions. Parameterization: $N_1 = 1000$, $\mu = 0.05$, $T = 4$, $R = 3$, $P = 1$, $S = 0$.

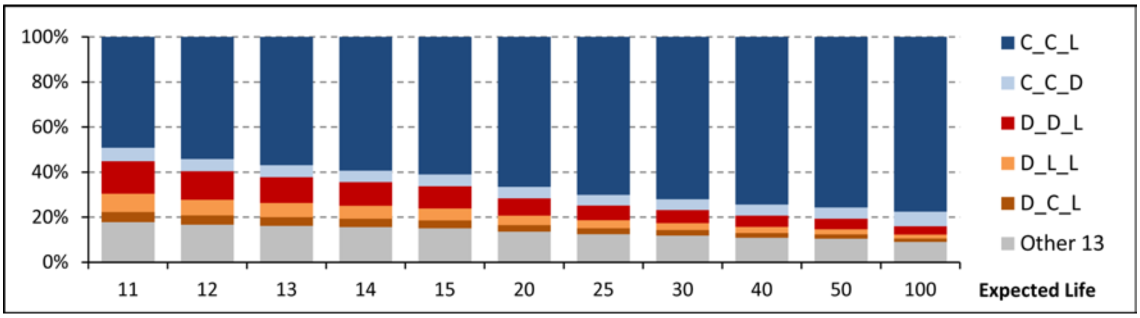

Fig. S4. Composition of strategies in the cooperative regime computed in Fig. S3.

Figures S3 and S4 can be replicated using the applet provided.

5.2 Mean-field approximation

The mean-field equations for the model where the population size is variable are identical to the mean-field equations for the model where the population size is constant. This statement rests on the following two observations:

a. In the dynamics of the model with variable population, the effect of one single unmatched individual, if his payoff is bounded, becomes negligible in large populations.

b. Both the number of births and the number of deaths as a fraction of the population size converge in probability to β as $N \to \infty$.