Mathematica as a teaching tool to illustrate the micro-macro link in the context of evolutionary game theory

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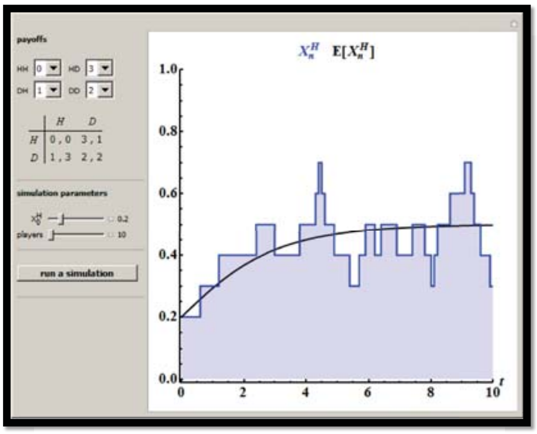
Many models in Economics are aggregate in the sense that they describe at the macro level dynamic processes which are actually the result of various assumptions at the micro level. In many such cases, the necessary step to go from the micro-foundations to the aggregate equations requires mean-field approximations that students often find difficult to grasp. *Mathematica* is an excellent tool to illustrate this micro-macro link, since it facilitates the implementation of simple computer models with which students can play and, in this way, see for themselves the conditions under which the mean-field approximations work best. The added value of *Mathematica* lies on two pillars:

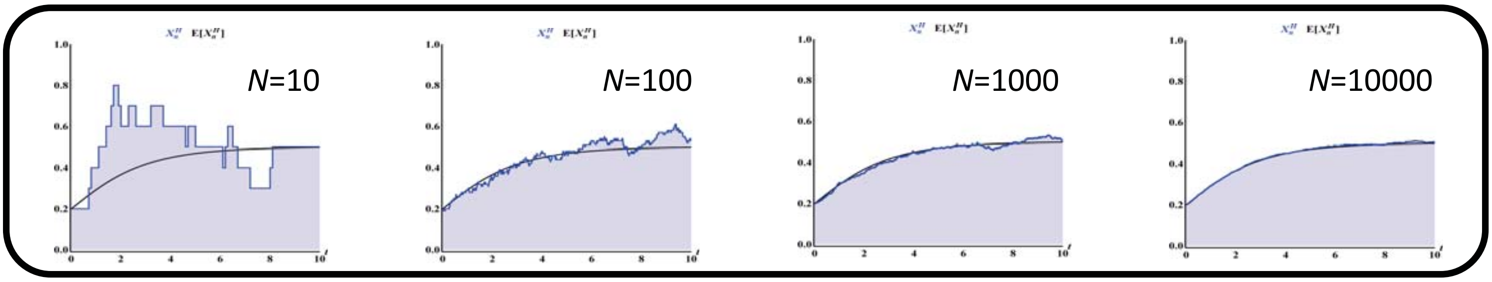
1. *Mathematica* allows for the simulation of stochastic processes and the simultaneous resolution of differential equations within the same framework. This facilitates the comparison of the two approaches in a graphical and interactive way.
2. *Mathematica* allows for the implementation of simple graphical interfaces, with which students can interact and explore the effect of changing different parameters in the model.

Expected Dynamics of an Imitation Model in 2x2 Symmetric Games [Link to the Program]

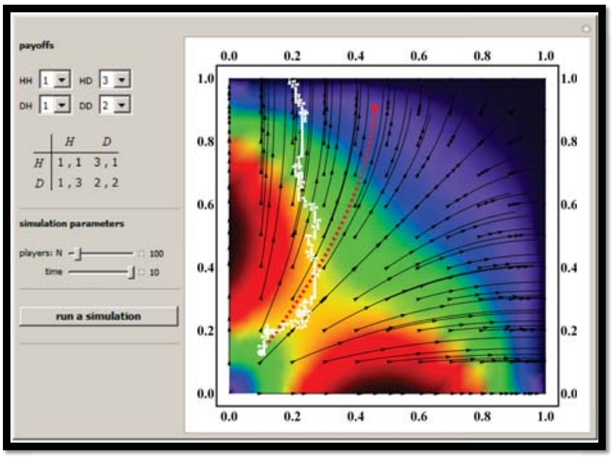


The figure in the *Mathematica* model shows the actual (in blue) and expected (in black) proportion of *H*-strategists in a population of *N* individuals who, at each iteration (of time length 1/*N*), are randomly matched in pairs to play a symmetric 2×2 game. The two possible actions (or pure strategies) in the game are labelled *H* and *D*. Thus, each individual in the population is either an *H*-strategist or a *D*-strategist. The payoffs of the game are HH, HD, DH, and DD (parameters), where, for instance, HD denotes the payoff obtained by an *H*-strategist when he plays with a *D*-strategist.

At the end of each iteration, after all individuals have played the game, one randomly selected player revises her strategy (*H* or *D*) according to the following rule: "I look at another (randomly selected) individual; if and only if she got a payoff higher than mine, I adopt her strategy".



Expected Dynamics of an Intra-Population Imitation Model for Inter-Population 2x2 Symmetric Games



Consider two distinct populations with the same number of individuals *N*. At each iteration (of time length 1/*N*), all individuals are randomly matched in pairs made up of one individual from each population to play a symmetric 2×2 game. The two possible actions (or pure strategies) in the game are labelled *H* and *D*. Thus, each individual (regardless of the population to which it belongs) is either an *H*-strategist or a *D*-strategist. The payoffs of the game are HH, HD, DH, and DD (parameters), where, for instance, HD denotes the payoff obtained by an *H*-strategist when he plays with a *D*-strategist.

At the end of each iteration, after all individuals have played the game, one randomly selected player from each population revises her strategy (*H* or *D*) according to the following rule: "I look at another (randomly selected) individual in my population; if and only if she got a payoff higher than mine, I adopt her strategy". Thus, the game is played between individuals of different populations, but imitation takes place within each population.

The figure in the *Mathematica* model shows a simulation of the proportion of *H*-strategists in each population (in white), its expected dynamics (in dashed red), and the phase plane of the expected dynamics (mean field) in the background.

[Link to the Program]