

List of figures

- Figure 4-1. Expected motion of the system in a Stag Hunt game parameterised as $[3, 4, 1, 0 | 0.5 | 0.5]^2$, together with a sample simulation run (40 iterations). The arrows represent the expected motion for various states of the system; the numbered balls show the state of the system after the indicated number of iterations in the sample run. The background is coloured using the norm of the expected motion. For any other learning rate the size of the arrows would vary but their direction would be preserved..... 59
- Figure 4-2. Trajectories in the phase plane of the differential equation corresponding to a Stag Hunt game parameterised as $[3, 4, 1, 0 | 0.5 | 0.5]^2$, together with a sample simulation run (40 iterations). The background is coloured using the norm of the expected motion. 60
- Figure 4-3. Trajectories in the phase plane of the differential equation corresponding to the Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | l]^2$, together with a sample simulation run ($l = 2^{-4}$). This system has a SCE at $[p_{1,C}, p_{2,C}] = [0.37, 0.37]$. The background is coloured using the norm of the expected motion. 61
- Figure 4-4. Solutions of differential equation [4-2] for the Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | l]^2$ with different (and symmetric) initial conditions $[p_{1,C}, p_{2,C}] = [x_0, x_0]$. This system has a unique SCE at $[p_{1,C}, p_{2,C}] = [0.37, 0.37]$ and a unique SRE at $[p_{1,C}, p_{2,C}] = [1, 1]$ 62
- Figure 4-5. Expected motion of the system in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | 0.5]^2$, with a sample simulation run. 64
- Figure 4-6 Figure showing the most likely movements at some states of the system in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | 0.5]^2$, with a sample simulation run. The background is coloured using the norm of the most likely movement..... 65
- Figure 4-7. Probability of starting an infinite chain of the Mutually Satisfactory (MS) outcome CC in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | l]^2$. The 5 different (coloured) series correspond to different learning rates l . The variable x_0 , represented in the horizontal axis, is the initial probability of cooperating for both players. 66
- Figure 4-8. Histograms representing the probability to cooperate for one player (both players' probabilities are identical) after n iterations, for different learning rates $l_i = l$, with $A_i = 2$, in a symmetric Prisoner's Dilemma with payoffs $[4, 3, 1, 0]$. Each

histogram has been calculated over 1,000 simulation runs. The initial probability for both players is 0.5. The significance of the gray arrows will be explained later in the text..... 68

Figure 4-9. Three sample runs of a system parameterised as $[4, 3, 1, 0 | 2 | l]^2$ for different values of n and l . The product $n \cdot l$ is the same for the three simulations; therefore, for low values of l , the state of the system at the end of the simulations tends to concentrate around the same point..... 71

Figure 4-10. Histograms representing the propensity to cooperate for one player (both players' propensities are identical) after 1,000,000 iterations (when the distribution is stable) for different levels of noise ($\epsilon_i = \epsilon$) in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | 0.25]^2$. Each histogram has been calculated over 1,000 simulation runs. 75

Figure 4-11. Representative time series of player 1's propensity to cooperate over time for the Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | 0.5]^2$ (left) and $[4, 3, 1, 0 | 2 | 0.25]^2$ (right), with initial conditions $[x_0, x_0] = [0.5, 0.5]$, both without noise (top) and with noise level $\epsilon_i = 10^{-3}$ (bottom). 76

Figure 4-12. Evolution of the average probability / propensity to cooperate of one of the players in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 2 | 0.5]^2$ with initial state $[0.5, 0.5]$, for different levels of noise ($\epsilon_i = \epsilon$). Each series has been calculated averaging over 100,000 simulation runs. The standard error of the represented averages is lower than $3 \cdot 10^{-3}$ in every case. 77

Figure 4-13. Evolution of the average probability / propensity to cooperate of one of the players in a Prisoner's Dilemma game parameterised as $[4, 3, 1, 0 | 0.5 | 0.5]^2$ with initial state $[0.9, 0.9]$, for different levels of noise ($\epsilon_i = \epsilon$). Each series has been calculated averaging over 10,000 simulation runs. The inset graph is a magnification of the first 500 iterations. The standard error of the represented averages is lower than 0.01 in every case. 78

Figure 4-14. One representative run of the system parameterised as $[4, 3, 1, 0 | 0.5 | 0.5]^2$ with initial state $[0.9, 0.9]$, and noise $\epsilon_i = \epsilon = 0.1$. This figure shows the evolution of the system in the phase plane of propensities to cooperate, while figure 15 below shows the evolution of player 1's propensity to cooperate over time for the same simulation run. 79

Figure 4-15. Time series of player 1's propensity to cooperate over time for the same simulation run displayed in Figure 4-14..... 79

Figure 5-1. Payoff matrix of the "Tragedy of the Commons game" for a particular agent. 90

Figure 5-2. UML activity diagram of the CBR decision making algorithm.....	93
Figure 5-3. Average cooperation rates when modelling two players with Memory Length ml and Aspiration Threshold AT , playing the PD. The average cooperation rate shows the probability of finding both Players cooperating once they have finished the learning period (<i>i.e.</i> when the run locks in to a cycle). The values represented for $ml = 1$ have been computed exactly. The rest of the values have been estimated by running the model 10,000 times with different random seeds. All standard errors are less than 0.5 %.....	97
Figure 5-4. Average cooperation rates when modelling two players with Memory Length ml , Aspiration Threshold greater than <i>Temptation</i> , and with 3 different representations of the state of the world (D1, D2, and D1&D2), playing the PD. The values represented for $ml = 1$ have been computed exactly. The rest of the values have been estimated by running the model 10,000 times ($ml = 2, 3, 4$) or 1,000 times ($ml = 5, 6$) with different random seeds. All standard errors are less than 1%.	101
Figure 5-5. Average reward rates for different values of M in the Tragedy of the Commons game played by 10 Players with Memory Length $ml = 1$. Each represented value has been estimated by running the model 1,000 times. All standard errors are less than 1.5%.	102
Figure 5-6. Average reward rates for different values of M in the Tragedy of the Commons game played by 10 Players with Memory Length $ml = 1$. Every player A can observe other 5 players only, who are the only ones that can observe player A . Each represented value has been estimated by running the model 1,000 times. All standard errors are less than 1.5%.	103
Figure 5-7. Elimination of dominated outcomes. Figure b shows the remaining outcomes after having applied one step of outcome dominance. Figure c shows the remaining outcomes after having applied two steps of outcome dominance. Red crosses represent outcomes which are unacceptable for player Red (row), blue crosses represent outcomes which are unacceptable for player Blue (column), and black crosses represent outcomes eliminated in previous steps.....	107
Figure 5-8. Average proportion of outcomes where both players are cooperating in the Prisoner's Dilemma (PD), calculated over 100 time-steps starting at time-step 1001, and using 500 simulation runs for each data point. The payoffs in the game are represented by its initial letter: S for Suckers, P for Punishment, R for Reward, and T for Temptation.	112

Figure 5-9. Stochastically stable outcomes (highlighted in white) in various 2-player 2-strategy games. Payoffs are numeric for the sake of clarity, but only their relative order for each player is relevant.....	115
Figure 6-1. Snapshot of the interface in EVO-2x2.....	123
Figure 6-2. Representation of players in the strategy space using EVO-2x2-3D (left) and EVO-2x2 (right). The image on the right shows the top-down projection of the representation on the left.....	127
Figure 6-3. Example of a graphical summary of the results obtained with EVO-2x2. This figure is automatically created and placed in the appropriate folder by the supporting scripts.....	128
Figure 6-4. Sketch showing the relationship between the 3D contour plot and the accompanying 2D density plots created by the supporting scripts.....	129
Figure 6-5. Accumulated frequency of different types of strategies in 8 simulation runs starting from different initial conditions. Axes are as in Figure 6-3.....	134
Figure 6-6. Influence of the mutation rate on the dynamics of the system. TFT measures the average time that strategies with $PC \geq (13/15)$, $PC/C \geq (13/15)$ and $PC/D \leq (2/15)$ were observed.....	135
Figure 6-7. Influence of the number of players in the population. TFT measures the average time that strategies with $PC \geq (13/15)$, $PC/C \geq (13/15)$ and $PC/D \leq (2/15)$ were observed.....	136
Figure 6-8. Influence of different pairing mechanisms. TFT measures the average time that strategies with $PC \geq (10/15)$, $PC/C \geq (10/15)$ and $PC/D \leq (5/15)$ were observed; ALLD measures the average time that strategies with $PC \leq (5/15)$, $PC/C \leq (5/15)$ and $PC/D \leq (5/15)$ were observed.....	137
Figure 6-9. Stochastic (mixed) strategies vs. deterministic (pure) strategies: influence in the system dynamics. TFT measures the average time that strategies with $PC \geq (10/15)$, $PC/C \geq (10/15)$ and $PC/D \leq (5/15)$ were observed; ALLD measures the average time that strategies with $PC \leq (5/15)$, $PC/C \leq (5/15)$ and $PC/D \leq (5/15)$ were observed.....	138
Figure 6-10. Influence in the distribution of outcomes (CC, CD/DC or DD) of augmenting the set of possible values for PC , PC/C and PC/D	138
Figure 6-11. Influence of augmenting the set of possible values for PC , PC/C and PC/D in the average values of these variables in the population.....	139